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Use the $\boxed{\text{Mean Value Theorem}}$ to determine where the slope of the secant line equals the slope of the tangent line

A) $f(x) = x^2$ [2,4] $x=2$ $x=4$

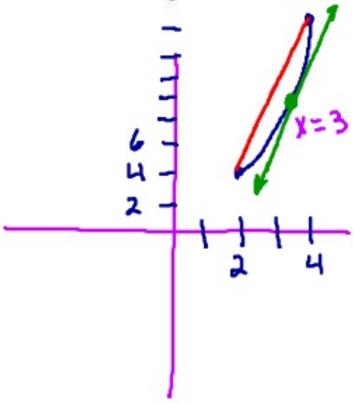
$(2,4)$ $(4,16)$

$f'(x) = 2x$

$m = \frac{16-4}{4-2} = 6$

$2x = 6$

$x = 3$



Is f(x) continuous from [1,8]?

Is f'(x) defined from (1,8)?

B) $f(x) = x^{\frac{1}{3}}$ [1,8]

$(1,1)$ $(8,2)$

$f'(x) = \frac{1}{3}x^{-2/3}$

$= \frac{1}{3\sqrt[3]{x^2}}$

$m = \frac{2-1}{8-1} = \frac{1}{7}$

$\frac{1}{3\sqrt[3]{x^2}} = \frac{1}{7}$

$3\sqrt[3]{x^2} = 7$

$\sqrt[3]{x^2} = \frac{7}{3}$

$x^2 = \left(\frac{7}{3}\right)^3$

$x = \pm\sqrt{\left(\frac{7}{3}\right)^3}$

C) $f(x) = x^{\frac{1}{3}}$ [0,1]

$(0,0)$ $(1,1)$ $m=1$

$\frac{1}{3\sqrt[3]{x^2}} = \frac{1}{1}$

D) $f(x) = x^2$ [-2,2]

$(-2,4)$ $(2,4)$

$m = 0$

$2x = 0$

$x = 0$

Rolle's Thm

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slope of tangent line $(f'(x))$

92. Let f be the function defined by f(x) = x + ln(x). What is the value of c for which the instantaneous rate of change of f at x = c is the same as the average rate of change of f over [2, 6]?

slope of secant

$f'(x) = 1 + \frac{1}{x}$

$(2, 2 + \ln 2)$ $(6, 6 + \ln 6)$

$m = \frac{2 + \ln 2 - (6 + \ln 6)}{2 - 6}$

$1 + \frac{1}{x} = \frac{-4 + \ln 2 - \ln 6}{-4}$

$m = \frac{-4 + \ln 2 - \ln 6}{-4}$

If $f(x) = \cos\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2} < x < \frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Find those values.

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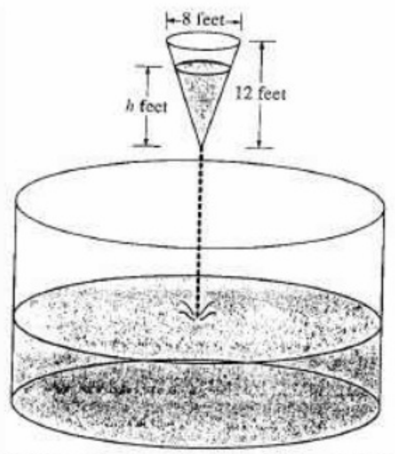
$\frac{r}{h} = \frac{4}{12}$ $\frac{r}{h} = \frac{1}{3}$ $r = \frac{1}{3}h$	$r = 4$ 21. Water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth, h, in feet, of the water in the conical tank is changing at the rate of (h - 12) feet per minute. Volume of a cone: $V = \frac{1}{3}\pi r^2 h$ $\pi r^2 = 400\pi$ $\frac{dh}{dt} = h - 12$  A) Write an expression for the volume of water in the conical tank as a function of h. $V = \frac{1}{3}\pi r^2 h$ $V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$ B) At what rate is the volume of water in the conical tank changing when h = 3? Indicate units of measure. $V = \frac{1}{27}\pi h^3$ $\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$ $\frac{dh}{dt} = h - 12$ $\frac{dV}{dt} = \frac{1}{9}\pi(3)^2(-9)\ \frac{ft^3}{min}$ $\frac{dh}{dt} = -9$ C) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when h = 3? Indicate units of measure. $V = \pi r^2 h$ $V = 400\pi y$ $9\pi = 400\pi \frac{dy}{dt}$ $V = (\pi r^2) y$ $\frac{dV}{dt} = 400\pi \frac{dy}{dt}$ $\boxed{\frac{dy}{dt} = \frac{9\pi}{400\pi}\ ft/min}$